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DLF
DYDAKTYCZNE LABORATORIUM FIZYCZNE

Experiment 40

Violation of Bell's inequality (CHSH) for polarisation-entangled photon pairs

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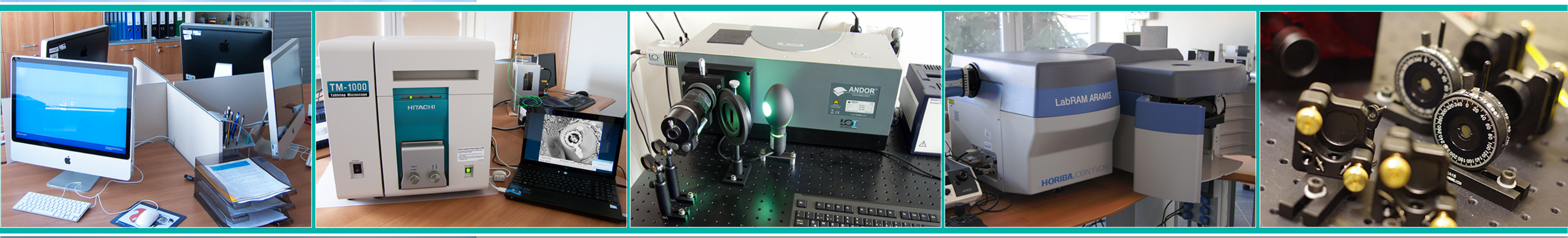
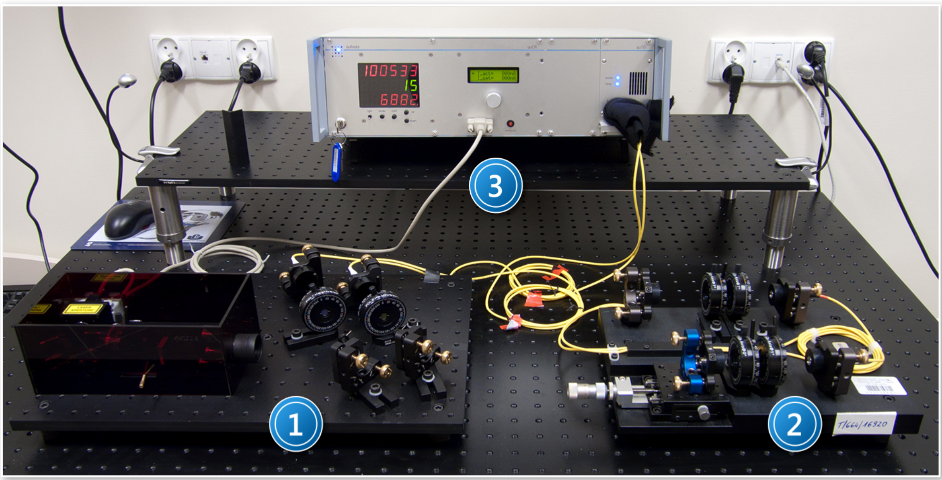


I. Background theory.

1. Description of composite systems in classical and quantum theory.
2. Background and derivation of the CHSH inequality.
3. Product and entangled states for bipartite quantum systems.
4. Description of photon polarisation states.
5. Bell states with polarised pairs of photons, violation of Bell's inequality by these states.
6. Physical Consequences of breaking the CHSH inequality.
7. Natural birefringence.
8. Operation of wave plates.
9. Light polarisers. Glan-Thompson polariser.
10. Parametric down-conversion.
11. Construction and operation of semiconductor lasers.

II. Experimental tasks.

1. Refer to the experimental setup shown in *Pictures 1* and *2* and in *Figure 3* in *Appendix A*.

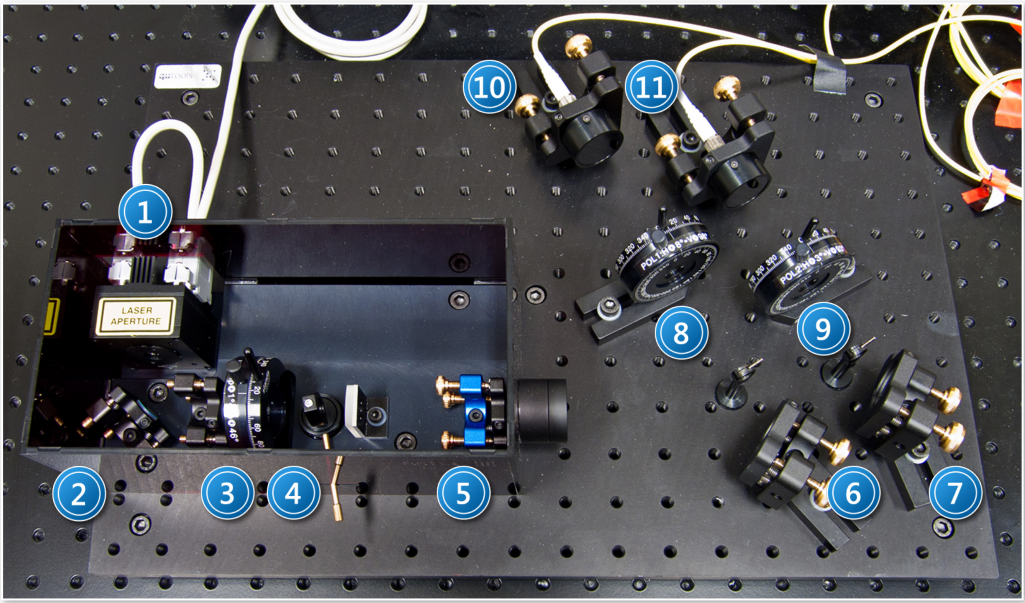


Picture 1. Experimental setup to study the properties of polarisation-entangled photon pairs: 1 – system for demonstrating entanglement visibility; 2 – system for studying two-photon interference; 3 – multi-function unit: power supply – controller – single-photon detector.

2. Test the procedures related to preparing the experimental setup for measurements and ensure that the geometrical arrangement of the optical elements is correct, as described in detail in *Appendix B*.
3. Upon completion of all preliminary steps described in *Appendix B,* choose a Bell state (equation (1)). Use the condition $\Phi = 0$.

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left[|H\rangle_1|H\rangle_2 + e^{i\phi}|V\rangle_1|V\rangle_2\right] \tag{1}$$

For a state such as $|\Phi^+\rangle$, the coincident counts should be maximum for both combinations of polariser settings (HH and VV) with a corresponding setting for the initial compensation crystal 4 in *Picture* 2.



Picture 2. quED I layout for demonstrating the quality of entangled photon pairs: 1 – laser; 2, 6, 7 – mirrors; 3 – half-wave plate; 4 – initial conpensation crystal (YVO_4); 5 – BBO crystal; 8, 9 – polarisers; 10, 11 – optical fibre with couplings.

4. Show experimental violation of Bell's inequality in entangled states.

Table 1. Table to test the Bell inequality.

Settings	Pol 1	Pol 2	C_1	C_2	C_3	C_4	C_5	$\bar{C}$	$\Delta\bar{C}$
$C(\alpha,\beta)$	0°	22,5°							
$C(\alpha,\beta_\perp)$	0°	112,5°							
$C(\alpha_\perp,\beta)$	90°	22,5°							
$C(\alpha_\perp,\beta_\perp)$	90°	112,5°							
$C(\alpha',\beta)$	45°	22,5°							
$C(\alpha',\beta_\perp)$	45°	112,5°							
$C(\alpha'_\perp,\beta)$	135°	22,5°							
$C(\alpha'_\perp,\beta_\perp)$	135°	112,5°							
$C(\alpha,\beta')$	0°	67,5°							
$C(\alpha,\beta'_\perp)$	0°	157,5°							
$C(\alpha_\perp,\beta')$	90°	67,5°							
$C(\alpha_\perp,\beta'_\perp)$	90°	157,5°							
$C(\alpha',\beta')$	45°	67,5°							
$C(\alpha',\beta'_\perp)$	45°	157,5°							
$C(\alpha'_\perp,\beta')$	135°	67,5°							
$C(\alpha'_\perp,\beta'_\perp)$	135°	157,5°							
							Sum	$\sum\bar{C}$	$\sum\Delta\bar{C}$

To do this, measure the number of coincidence counts (on the single photon detector display), successively for the polariser settings listed in Table 1.

In order to minimise the effect of detector fluctuations, record the number of counts five times for each set of polarisers.

Hint

When setting the polarisers, take their mountings:

for polariser Pol 1 (8 in *Picture 2*), position H corresponds to 8°; for polariser Pol 2 (9 in *Picture 2*), position H corresponds to 3°.

Complete Table 1 by using all 16 possible configurations for the polarisers.

5. The average $\bar{C}$ of the five readings can be calculated with equation (2), setting n = 5 ,

$$\bar{C}(\alpha,\beta) = \frac{1}{n}\sum_{j=1}^{n} C_j\,(\alpha,\beta), \tag{2}$$

where C_j is the result of a single reading, and the mean square error $\Delta\bar{C}(\alpha,\beta)$, is calculated from the formula in equation (3),

$$\Delta\bar{C}(\alpha,\beta) = \sqrt{\frac{\sum_{j=1}^{n}\left[C_j(\alpha,\beta) - \bar{C}(\alpha,\beta)\right]^2}{n(n-1)}} \tag{3}$$

Fill in the appropriate fields in Table 1.

6. Calculate the sum $\sum\bar{C}$ (for all settings of the polarisers), the average value of counts and the total mean square error $\sum\Delta\bar{C}$ and enter them into Table 1.
7. Calculate the Bell parameter using the formula in equation (4)

$$S(\alpha,\alpha',\beta,\beta') = E(\alpha,\beta) + E(\alpha',\beta) - E(\alpha,\beta') + E(\alpha',\beta'), \tag{4}$$

where the correlations $E(\alpha,\beta)$ are expressed by the number of coincidences as in equation (5)

$$E(\alpha,\beta) = \frac{\bar{C}(\alpha,\beta) - \bar{C}(\alpha,\beta_\perp) - \bar{C}(\alpha_\perp,\beta) + \bar{C}(\alpha_\perp,\beta_\perp)}{\bar{C}(\alpha,\beta) + \bar{C}(\alpha,\beta_\perp) + \bar{C}(\alpha_\perp,\beta) + \bar{C}(\alpha_\perp,\beta_\perp)}. \tag{5}$$

8. Estimate the resulting error in the experimental values for the Bell parameter S by using the approximate formula in equation (6)

$$\Delta S \cong \frac{\sum\Delta\bar{C}}{\sum\bar{C}}. \tag{6}$$

9. Compare your values of the Bell parameter with the data given in the quED I specifications given in *Appendix C.*

III. Apparatus.

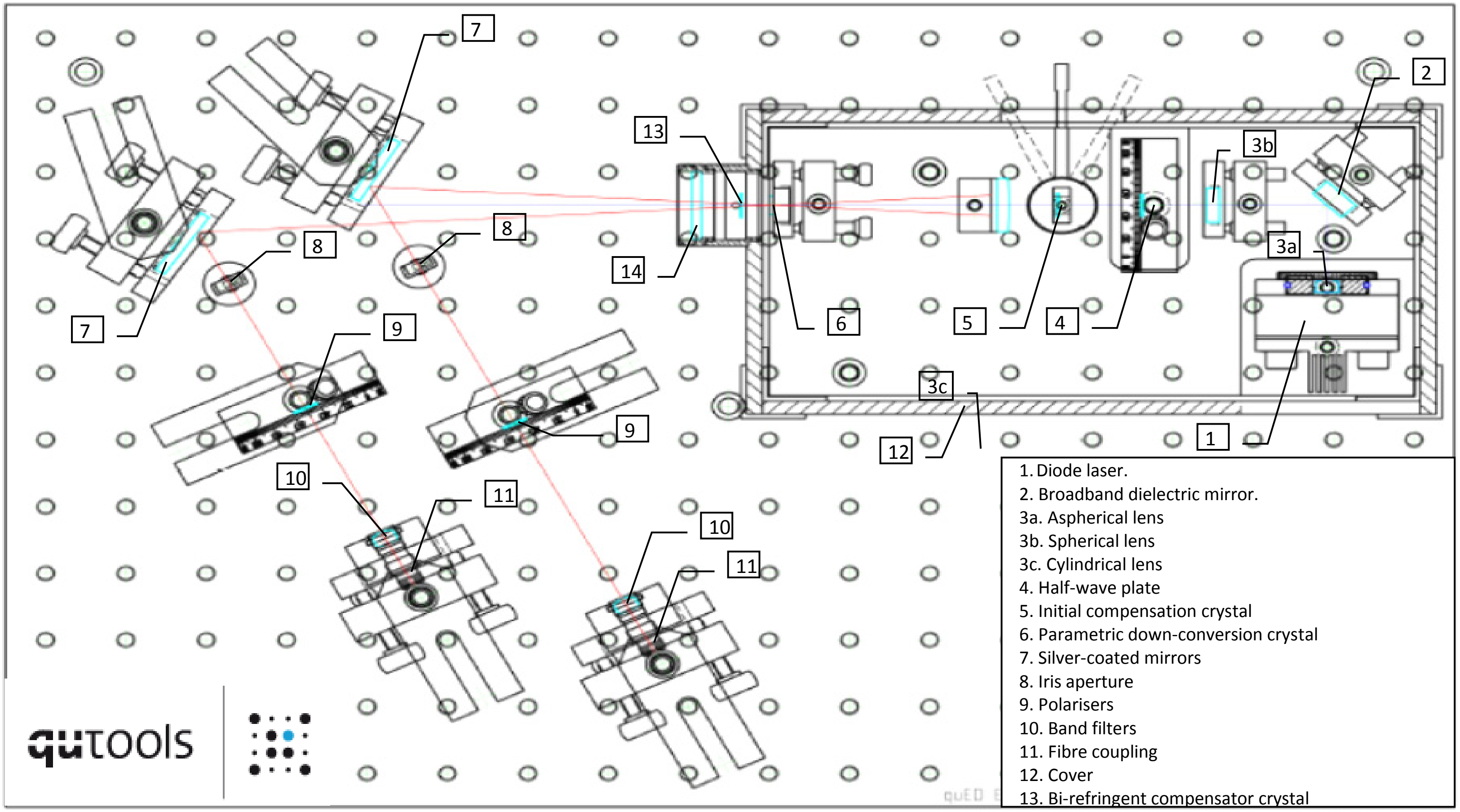
1. Semiconductor laser (λ = 401.5 nm, 10 mW).
2. 3 mirrors.
3. Lenses.
4. Half-wave plate.
5. 2 polarisers.
6. Bi-refringent crystal YVO_4.
7. Non-linear crystal BBO (β - BaB_2O_4).
8. Optical fibre couplers.
9. Single-mode fibre.
10. Band filters.
11. Single-photon detector.

IV. Literature.

1. M. Alicka, R. Alicki – *„Pracownia Informacji Kwantowej / Quantum Information Laboratory",* skrypt Uniwersytetu Gdańskiego, 2011.
2. M. Nielsen, I. Chuang – *"Quantum Computation and Quantum Communication"*, Cambridge, London 2000.
3. L. Mandel, E. Wolf – "*Optical Coherence and Quantum Optics*", Cambridge 1995.
4. D. Greenberger, A. Zeilinger – "*Physics World*", **8,** Nr 9, 33 (1995).
5. W. A. Shurcliff, S.S. Ballard – "*Polarized Light*", Princeton 1964.
6. M. Born, E. Wolf – "*Principles of Optics*", Cambridge University Press, Cambridge 1999.
7. O. Svelto – "*Principles of Lasers*", Plenum, New York 1998.
8. W.S.C. Chang – "*Principles of Lasers and Optics*", Cambridge University Press, 2005.
9. J.A. Buck – "*Fundamentals of Optical Fibres*", NJ: Wiley – Interscience, Hoboken, 2004.
10. A. Peres – "*Quantum Theory: Concepts and Methods*", Kluwer Academic Publishers, 1993.
11. W. Demtröder – "*Atoms, Molecules and Photons: an Introduction to Atomic - , Molecular – and Quantum – Physics*", Springer, Berlin 2006.
12. D. Dehlinger, M.W. Mitchell – "*Entangled photon apparatus for the undergraduate laboratory",* Am. J. Phys. **70**, 989 – 901 (2002).
13. S. Nakamura, G. Fasol – "*The blue laser diode*", Springer, Heidelberg 1997.
14. H. Paul – "*Introduction Quantum Optics from Light Quanta to Teleportation*", Cambridge University Press, Cambridge 2004.
15. M. Le Bellac – "*A Short Introduction to Quantum Information and Quantum Computation*", Cambridge University Press, Cambridge 2006.

Appendix A

Figure 3. Schematic overview for the experimental setup to test pairs of polarisation-entangled photons



Appendix B

Preparing the experimental setup for measurements.

1. The geometry of the optical setup that provides equal amounts of photons with polarization H (horizontal) and V (perpendicular) recorded by the detector on both single-photon counting channels (as in *Figure 4*) requires setting the half-wave plate (3 in *Picture 2*) to 23,5°. Check that this is the case.

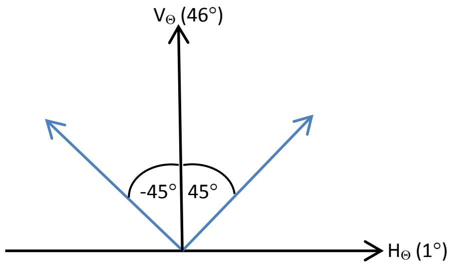
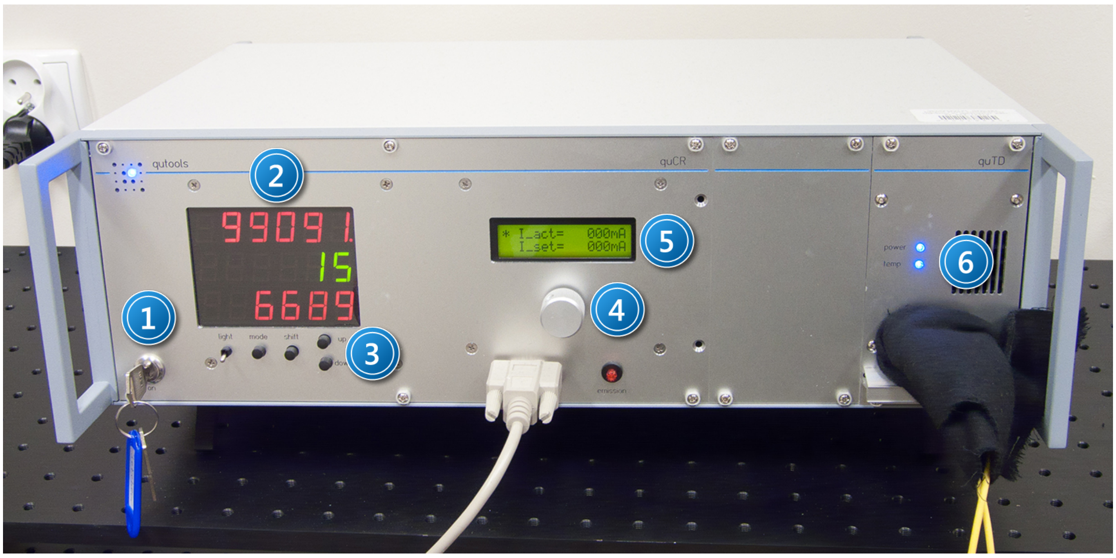


Figure 4. The direction of the pumping beam to ensure an equal number of photons with polarization H and V (with angle 23,5° for the half-wave plate 3 in Picture 2).

2. Turn on the single-photon detector (by turning the key 1 in *Picture 5* and *Figure 6*).



Picture 5. Measuring unit – controller, multifunction power supply, single-photon detector: 1 – main switch; 2 – count display: 3 – count function keys; 4 – laser settings dial; 5 – laser settings display; 6 – APD module indicators.

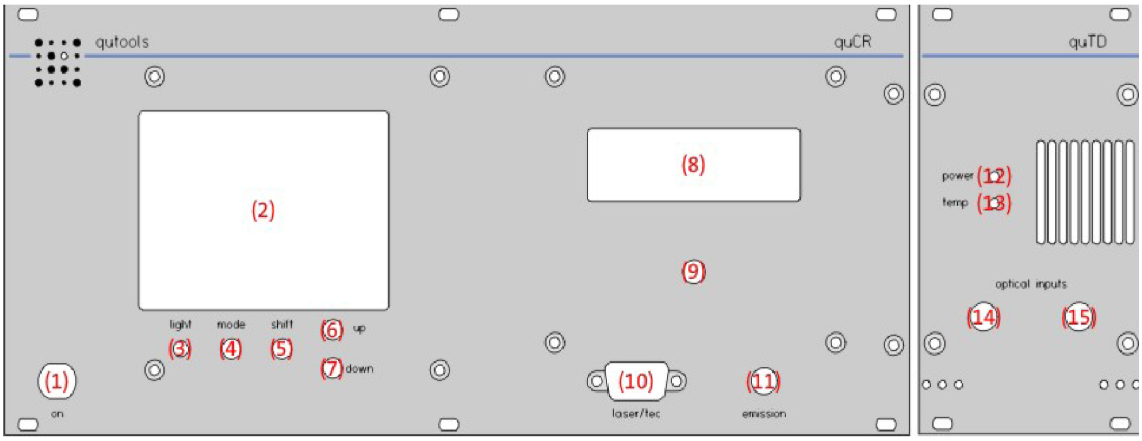


Figure 6. Schematic overview of the measuring unit front panel.

3. Wait until the blue temperature LED goes off (13 in *Figure 6*) on the detector front panel. This is equivalent to fixing the temperature of the cooling laser to the desired value of -30 ^{0}C.
4. Check the display 5 in *Picture 5* (8 in *Figure 6*) to see whether temperature T set is around 22 ^{0}C (this value was set in the programming). If this is not the case, adjust the temperature with the temperature knob 4 in *Picture 5* (9 in *Figure 6*).
5. Set the pumping laser power supply's current I (operating current) to 38 mA (with dial 3 in *Figure 6*).
6. Set the orientation for both polarisers 8 and 9 in *Picture 2* to H (horizontal).
7. Set the integration time to 5 seconds with knob 7 in *Figure 6* (to reduce fluctuations).
8. Measure coincidences (on the detector's central green display 2, *Figure 6*) for the combinations of polariser settings specified in step II.4.

Hint

The green numbers on the detector display indicate the number of coincidences; the red upper and lower numbers show the number of single photons in the specified integration period for each APD* channel.

* APD stands for Avalanche Photo Diode – the diode uses an avalanche effect in Geiger mode to count the number of single photons in the measuring channel.

Appendix C

quED I experimental set certificate for demonstrating the properties of polarisation-entangled photon pairs

qutools

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Inspection Data Sheet

Built For: Instytut Fizyki Doswiadczalnej, Uniwersytet Gdanski
Ul. Wita Stwosza 57
80-952 Gdansk
Poland

Order No.: 14/4A/A120/2010

Model: quED I

Description: Entanglement Demonstrator - Source of Polarization Entangled Photon Pairs

Detected characteristics:

Dark-count rates	≈ 2400 s^{-1} and ≈ 2600 s^{-1}
Single-count rates*	≈ 45000 s^{-1} and ≈ 38000 s^{-1}
Coincidence-count rate*	≈ 4000 s^{-1}
Entanglement⋄	horizontal/vertical base: $\approx \mathbf{98}$ % diagonal base: $\approx \mathbf{95}$ %
Bell (CHSH) inequality†	horizontal/vertical base: $\mathbf{2.75 \pm 0.01}$

* laser diode at op. current, irises fully opened, polarizers removed from the setup

⋄ measured as the visibility of correlation curves in two complementary bases (not corrected for accidental coincidence detections)

⋄ integration time of 5 s per angular setting of polarizers (not corrected for accidental coincidence detections)

Basic Specifications:

Pump Laser Diode	threshold current (I_{act}): 28 mA operating current (I_{act}): 38 mA optical power: 10 mW ($< 0.2\mu$W after pump-beam protective enclosure) peak wavelength: 401.5 nm operating temperature: 25 °C

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Conversion Crystals	material: beta-barium borate (BBO, β-BaB_2O_4) phase matching: type I at $\theta = 29.3°$ ($\phi = 0.0°$) dimensions: twice $7 \times 7 \times 0.7$ mm^3
Compensation Crystals	material: undoped yttrium orthovanadate (YVO_4)
Longpass Filters	material: Schott RG715 transmission @ op. wavelength: ≈ 91 %
Polarizers	contrast: $> 1000 : 1$ transmission @ op. wavelength: > 80 %
Source Dimensions	$30 \times 45 \times 10$ cm^3

Approved by: Pavel Trojek

Date: 13.04.2010

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